

**Guideline**

## The 5-step Hyp. Test (Short Version)

Step 1 State  $H_0$  and  $H_A$

Step 2 Conditions Check

Step 3 Test statistic

Step 4 P-value AND decision to reject  $H_0$   
or fail to reject  $H_0$ .

Step 5 Full sentence conclusion that  
interprets the decision made in Step 4  
in the context of the problem.

The longer version is after this page  
You may use this handout on the test and final exam.

# How to do a hyp. test

**Step 1** Write the null and alt. hypotheses.

The hypotheses should be mathematical statements involving one of the parameters  $P$ ,  $\gamma$ , or  $\sigma$ , for example. Also the null hyp ( $H_0$ ) has to have the equals symbol ( $=$  or  $\leq$  or  $\geq$ ) in it. The alt. hyp is usually the claim given in the statement of the word problem. Also, the same number must be used in both hypotheses: for example

$$H_0: P = 0.15$$
$$H_A: P < 0.139$$

is incorrect because  
both these numbers should be the same!

## Step 1)

How to translate the claim  
into the  $H_A$  or  $H_0$

\* Most problems have the claim as the  $H_A$ .

\* The  $H_0$  can only have one of these 3  
symbols in it:  $=, \leq, \geq$

( $H_0$  always has the "equals case")

\* The  $H_A$  can only have one of these 3  
symbols in it:  $\neq, <, >$

First,

Write the claim as a hypothesis  
 $(p \leq 0.5) \text{ or } (n < 340 \text{ min})$

Then, if the claim has  $=, \leq, \text{ or } \geq$   
it's  $H_0$ , but if it has  $\neq, < \text{ or } >$ ,  
then your claim is  $H_A$

Step 1

Write the claim  
in symbolic form

$$p = 0.5$$
$$p < 0.35$$

Does the claim  
have either  $=$ ,  $\leq$ ,  
or  $\geq$  ??

yes

Claim  
is  $H_0$

↓ no

Claim  
is  $H_A$

	Two-tailed test	left-tailed test	Right-tailed test
The null hyp., $H_0$	$=$	$= \text{ or } \geq$	$= \text{ or } \leq$
The Alt. hyp., $H_A$	$\neq$	$<$	$>$

How to tell if you have  
a left-tailed, right-tailed, or  
2-tailed test.

useful in steps 1 & 4.

## Hypothesis Testing Conditions Check

### Step 2

Check to see if the sampling dist is normal. The way to do this is to check if there are at least 10 successes and 10 failures in the sample.

Another way to check this is to check if both  $n \cdot p_0$  and  $n \cdot (1-p_0)$  are both  $\geq 10$ . (Where  $p_0$  is the percent listed in  $H_0$  and  $H_A$ .)

The other condition to check is:

Is the sample Random or representative of the population?

If either of the 2 conditions above are not valid (satisfied) then we cannot conduct a hyp. test!! As no meaningful result will come from doing so

(Step 2 side note)

If both conditions from step 2 are true, then we have evidence that the sampling dist. is approx. normal, and we assume the sampling dist. curve is centered at the value stated in the null hyp. Because the sample is random or representative, we will ~~then~~ be making a valid scientific conclusion when we do the hyp test  
*(although nothing is proven absolutely)*

**Step 3** Find the value of the test statistic. The test statistic is the Z-Score of your sample statistic, given by the formula

$$Z = \frac{(\text{Sample stat}) - (\text{Value stated in the } H_0)}{(\text{St dev of the sampling distribution})}$$

## Step 4

Find the p-value, and use the p-value to make a decision as to whether or not to reject  $H_0$ .

If the p-value  $\leq \alpha$ , reject  $H_0$

If the p-val  $> \alpha$ , fail to reject  $H_0$ .

### Case I (Left-tailed test)

If the  $H_A$  has a less than symbol, then you have a left-tailed test AND the p-value is the prob. of getting a sample statistic  $<$  the one we got, assuming  $H_0$  is correct.

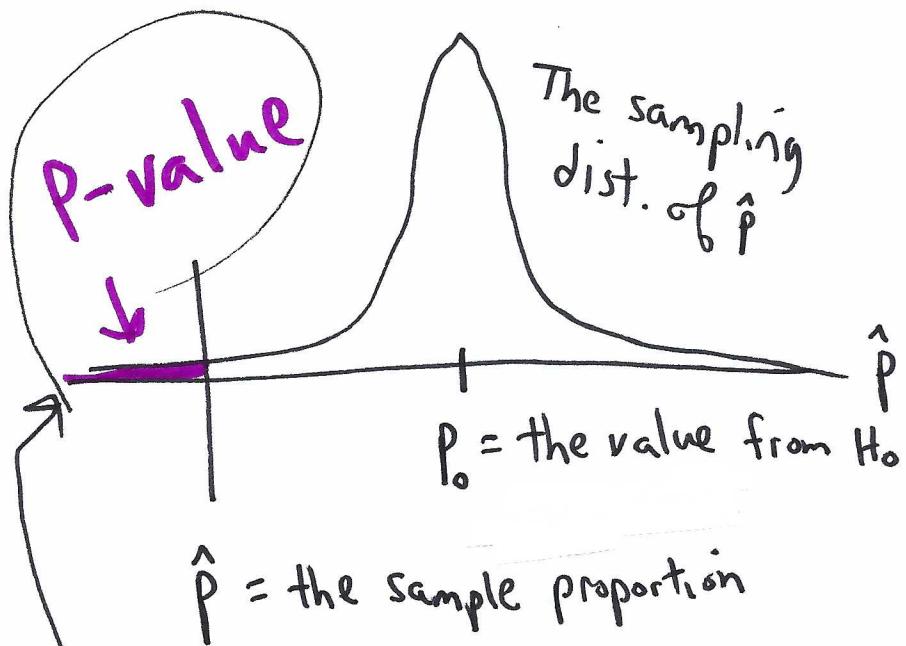
Moreover, the p-value is also equal to the prob. that  $Z <$  test statistic. In this case the p-val is the area under the sampling dist. graph that is left of  $\hat{p}$ , or equivalently, the area under the standard normal distribution that is left of the test statistic.

## Case I (Left-tailed test)

Step 4  
(continued)

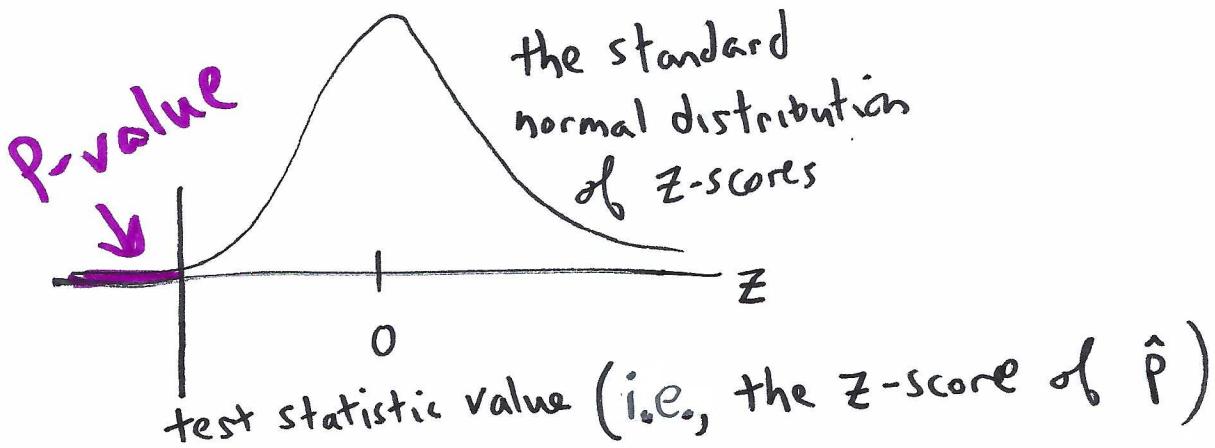
Here is a picture of the p-value

for the  
left-tailed test.



The p-value =  $P(\hat{p} < \text{the sample proportion}, \text{assuming } H_0 \text{ is correct})$

The p-value is also equal to  $P(Z < \text{the test statistic})$



Step 4

## Case 2 (the right-tailed test)

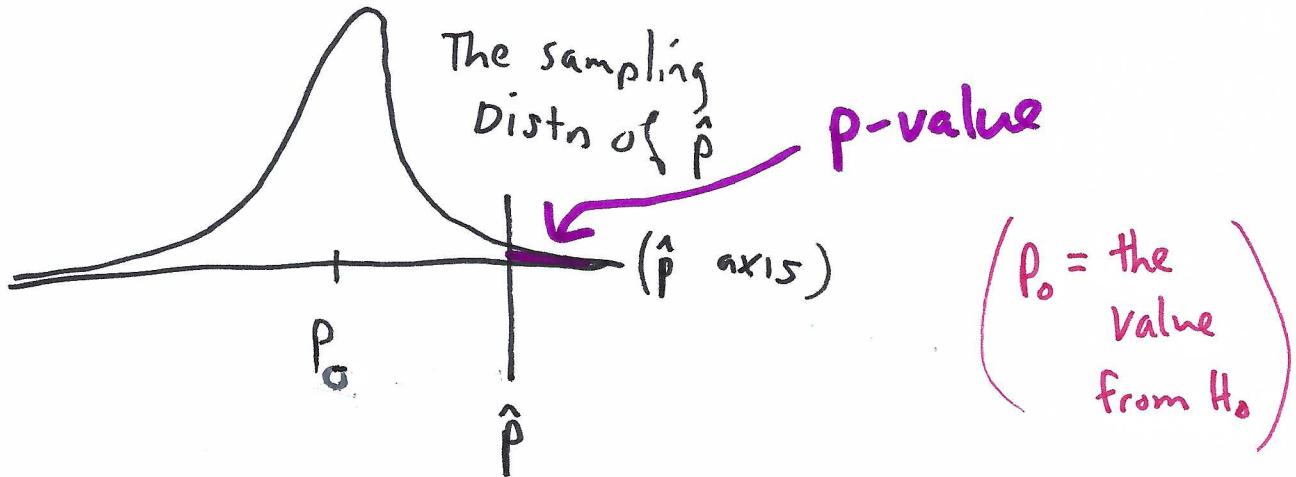
If the  $H_A$  has a greater than symbol, then you have a right-tailed test. The p-value is then the probability of getting a sample statistic greater than the one we got, assuming  $H_0$  is correct. Moreover, the p-value is also equal to the probability that  $Z > \text{the test statistic}$ .

In this case, the p-val is the area under the sampling dist. graph that is right of  $\hat{p}$ , or equivalently, the area under the standard normal dist. that is right of the test statistic

## Case 2 (Right-tailed test)

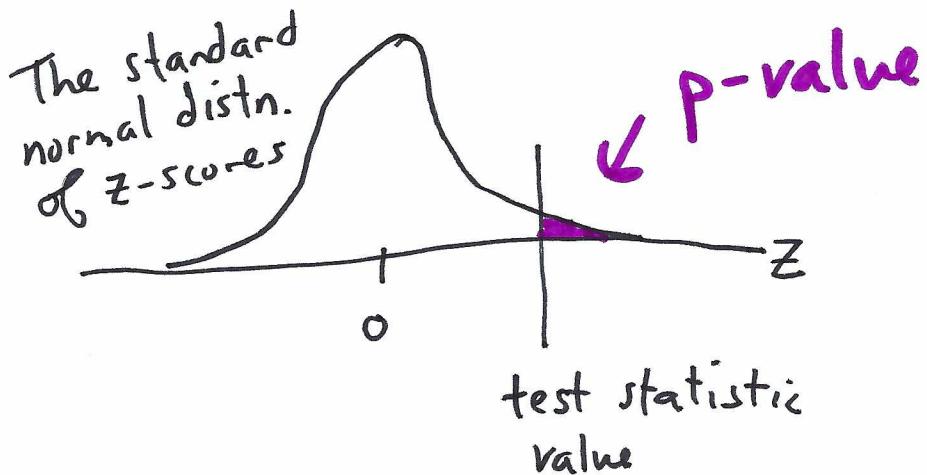
(Step 4  
Continued)

Here is a picture of the p-value for the right-tailed test.



The p-value =  $P(\hat{p} > \text{the sample proportion, assuming } H_0 \text{ is correct})$

The p-value is also equal to  $P(z > \text{the test statistic})$



## Step 4

### Case 3: the 2-tailed test

If the  $H_A$  has the  $\neq$  symbol, then you have a 2-tailed test. There are 2 types of two tailed tests:

- (1) When the sample statistic is left of the center of the sampling distr., or equivalently, when the test statistic is negative, AND
- (2) When the sample statistic is right of the center of the sampling distr., or equivalently, when the test statistic is positive.

$$(1) \text{ The p-value} = 2 \cdot P\left(\hat{p} < \text{the sample proportion, assuming } H_0 \text{ is correct}\right)$$
$$= 2 \cdot P(z < \text{the test statistic})$$

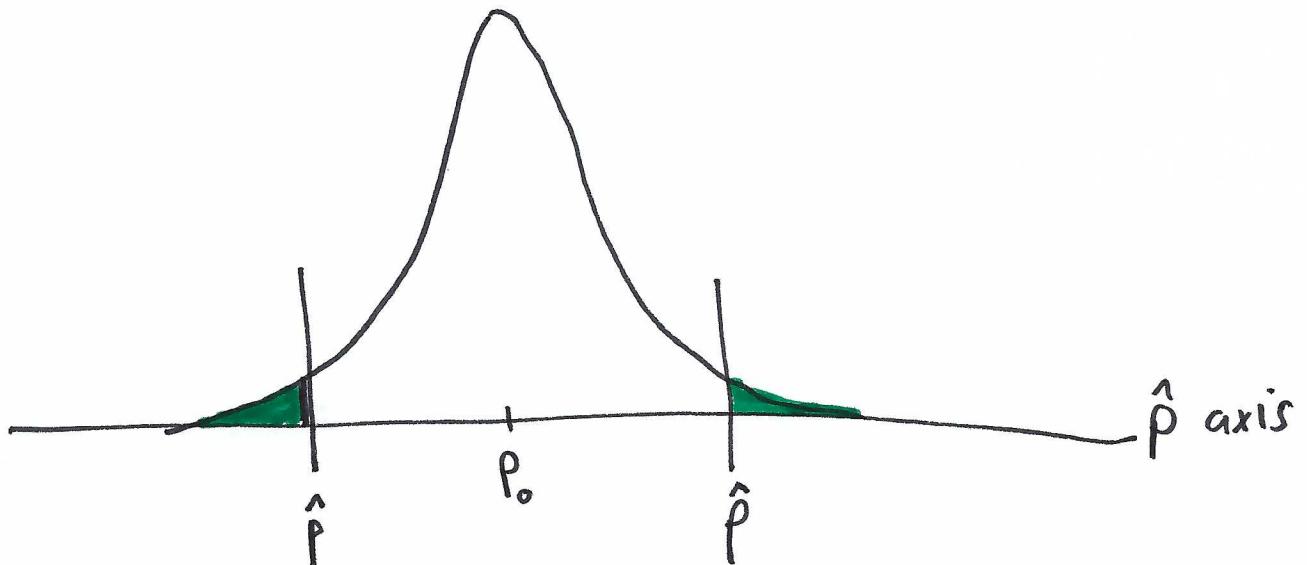
$$(2) \text{ The p-value} = 2 \cdot P\left(\hat{p} > \text{the sample proportion, assuming } H_0 \text{ is correct}\right)$$
$$= 2 \cdot P(z > \text{the test statistic})$$

I Always let the calculator find the p-value for me!

### Case 3 (2-tailed test)

Step 4

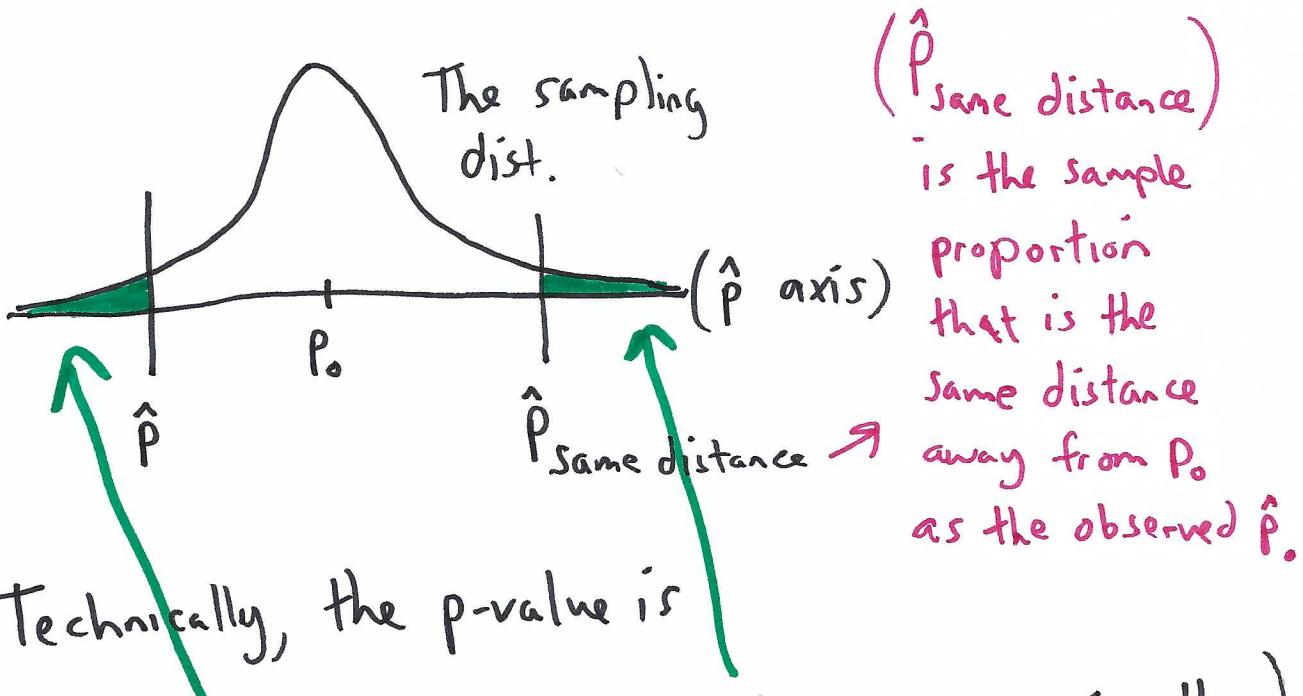
The 2-tailed test is used to detect sample proportions in either tail of the sampling distribution graph, which would be surprising.



Case 3 (2-tailed test)

(Step 4  
continued)

Here is a picture of the p-value for (1).



$(\hat{P}_{\text{same distance}})$   
is the sample proportion that is the same distance away from  $P_0$  as the observed  $\hat{P}$ .

And since  $P(A \text{ or } B) = P(A) + P(B)$  when A and B are mutually exclusive, this probability is

equal

$$P(\hat{P} < \text{the sample statistic}) + P(\hat{P} > \hat{P}_{\text{same distance}})$$

But because of symmetry, the area of one tail equals the area of the other tail,

So this last probability is equal to

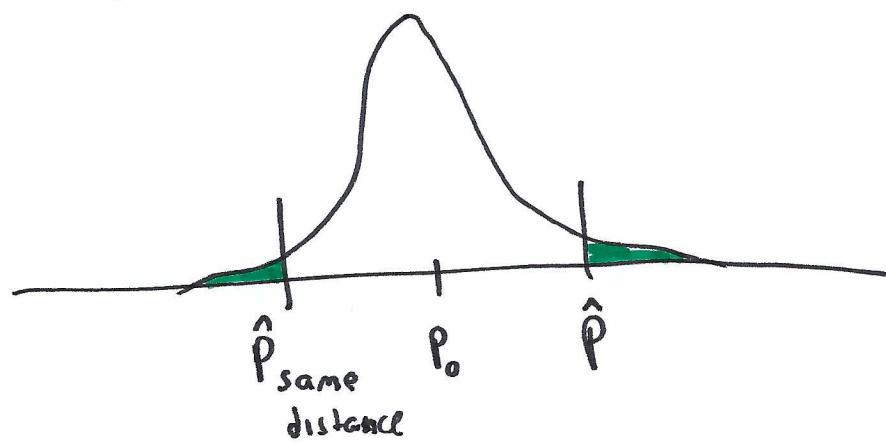
$$2 \cdot P(\hat{p} < \text{the observed sample proportion}, \text{ assuming } H_0 \text{ is correct})$$

Step 4

but this is also equal to

$$2 \cdot P(Z < \text{the test statistic})$$

For (2), the p-value is



$$P(\hat{p} < \hat{p}_{\text{same distance}} \text{ OR } \hat{p} > \text{the observed sample proportion}, \text{ assuming } H_0 \text{ is correct})$$

or equivalently,

$$2 \cdot P(\hat{p} > \text{the observed sample prop}), \text{ or equivalently, } 2 \cdot P(Z > \text{the test statistic})$$

Every hyp. test has you make a decision as to whether or not to reject  $H_0$ .

Guideline: How to interpret your decision

## Step 5

① If you decide to reject  $H_0$ , then the interpretation is:

"There is  $\downarrow$  convincing sample evidence to support [cut/copy/paste the  $H_A$ ]."

② If you decide to fail to reject  $H_0$  then the conclusion is:

". There is not convincing sample evidence to support [cut/copy/paste the  $H_A$ ]"